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Detail Preserving Sorted Difference Filter

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Abstract

A detail preserving filter that uses the intensity differences between a pixel and its neighbor pixels to eliminate impulse noises with minor variations is proposed. The absolute values of the intensity differences are sorted into a sequence in ascending order and the value at a specific position is used to determine whether the pixel under processing is an impulse noise or not. The method to find the specific position is provided and its feasibility is discussed. Theoretically impulse noises can be correctly selected when the density of the noises are not heavy. The filter preserves more fine details than the standard median filter in general. In the situation when the distribution of impulse noises has minor variations, the filter works better than the commonly used adaptive median filter and also produces cleaner results.

Keywords: *Sorted difference filter, median filter, adaptive median filter, impulse noises, detail preserving filter.*

1 Introduction

The standard median filter replaces the intensity value of a pixel being processed with the median value of the intensities in its neighborhood. The filter works very well on eliminating impulse noises, such as salt-and-pepper noises, when the density of the noises is not heavy ([1], [3], [11]). Impulse noises usually have a unipolar or bipolar distribution of intensities at one end or the two ends of the intensity range. In an image corrupted by impulse noises, white dots appearing in dark regions are called salts and dark dots appearing in bright regions are called peppers. There are quite a few impulse noise models adopted in research ([5]), among which the one with the following probability density function is widely used.

$$p(z) = \begin{cases} P_1, & \text{pepper; } z = 0, \\ P_2, & \text{salt; } z = L-1, \\ 1 - P_1 - P_2, & \text{noise free,} \end{cases} \tag{1}$$

where the range of intensities is the interval $[0, L-1]$, and P_1 and P_2 are the corresponding probabilities for peppers and salts, usually called densities. When $P_1 = P_2$, the noise is called a salt-and-pepper noise, and usually $P = P_1 + P_2$ refers to its total density. Because the intensities of peppers and salts are on the two ends of the intensity range of a corrupted image, they can be removed by the standard median filter effectively when P is low. Experimentally, when $P \leq 20\%$, the performance of the standard median filter is perceptually satisfactory.

In fact, the standard median filter still works well even though the noises are not perfect impulses, which display minor variations around the impulses in the distributions. The probability density function of these type of noises can be expressed as

$$p(z) = \begin{cases} p_1(z), & \text{pepper; } z \in [0, \epsilon_1], \\ p_2(z), & \text{salt; } z \in [L-1-\epsilon_2, L-1], \\ p_3(z), & \text{noise free,} \end{cases} \tag{2}$$

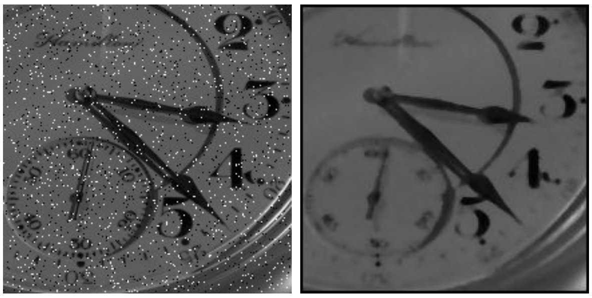
where the small tolerances ϵ_1 and ϵ_2 give two narrow intervals at the two ends of the intensity range $[0, L-1]$, in which noises occur with probabilities described by the functions $p_1(z)$ and $p_2(z)$, and $p_3(z) = 1 - p_1(z) - p_2(z)$ is the probability that a pixel with the intensity z is noise free. Denote

$$P = \int_0^{\epsilon_1} p_1(z)dz + \int_{L-1-\epsilon_2}^{L-1} p_2(z)dz$$

the total density of the impulse noise with minor variations. The shapes of the functions $p_1(z)$ and $p_2(z)$ are not the main concern of the performance of the standard median filter if ϵ_1 and ϵ_2 are relatively small.

However, the standard median filter is not detail preserving, with disadvantages including signal weakening and non-noisy image pixel corruption ([6]). This is because the intensity of a pixel usually is not exactly the median value in its neighborhood covered by the filter mask and it is altered during the process. More specifically, tiny components of objects in an image are usually eroded by the filter. For example, Figure

1 shows an image corrupted by salt-and-pepper noises with a total density $P = 10$ and the result processed by the standard median filter with size 5 by 5. In the resultant image, although the noises are removed, the fine details are heavily eroded and blurred by the filter.



(a) (b)

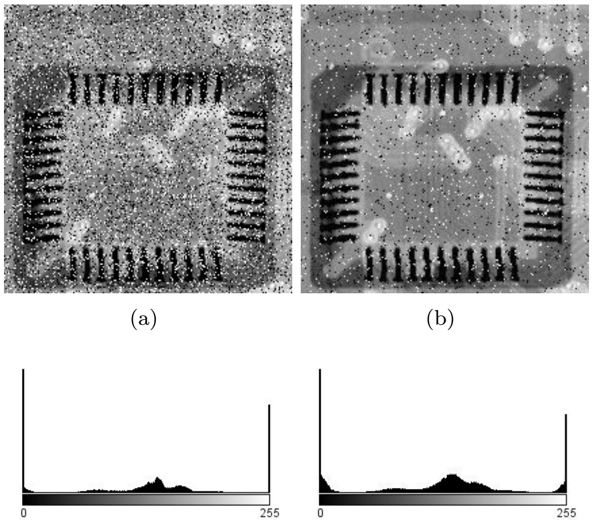
Figure 1: (a) A test image corrupted by salt-and-pepper noises with density 10%. (b) The result by the median filter with size 5 by 5.

Adaptive median filters are more efficient in detail preserving than the standard median filter on images with perfect impulse noises ([2], [3], [4], and [9]). With these filters each pixel is examined and classified as either a noise or a noise free pixel. A designated noise is then processed with the standard median filter or a modified one. For examples, a truncated median filter is used to process designated noises in ([7]); a decision based filter using multiple thresholds with multiple neighborhood information of the center pixel in the filter window is proposed to restore images corrupted by salt and pepper impulse noise ([8]); with modified median filters, decision making filtering technique can be used together with adaptive filters to improve efficiency ([10]).

The essence of the design of an adaptive median filter is on improving the accuracy of noise identification. We use the adaptive median filter in [3] as an example to explain the concept. The filter changes the size of its mask to find a proper median intensity value in a neighborhood of a pixel. If in the same neighborhood the intensity value of the pixel being processed is not an extreme value, its intensity value is not changed, and otherwise it is replaced with the median intensity value. The method tries not to replace the intensity value of a pixel unless it has to do so, in which case either the pixel has an extreme value, which is a candidate of an impulse noise, or the filter reaches its maximum size and the standard median filter must be used.

Generally, adaptive median filters work with the assumption that noises are impulses without variations, usually taking the two extreme intensity values 0 and $L - 1$ in the intensity range $[0, L - 1]$. Unfortunately, the perfect pattern does not always present in applications. In many situations the distribution of the noise intensities shows minor variations around the impulses, which frequently occur when the images are stored with compression, such as in JPEG format. In these situations adaptive median filters may not remove all the noises.

For example, Figure 2 shows an image corrupted by salt-and-pepper noises with minor variations on the impulses and the result obtained by the Adaptive median filter with maximum size 5 x 5. The test image was chopped from a test image used in [3], originally saved in TIFF format but was converted to JPEG format. In the TIFF format, the intensity distribution of the image displays a perfect pattern of two impulses at the two ends, while in the JPEG format, it shows two bumps at the two ends of the distribution, implying the noises are not perfect impulses. In the JPEG image the noises cannot be completely removed by the adaptive median filter.



(a) (b) (c) (d)

Figure 2: (a) An image corrupted by salt-and-pepper noises. (b) The result of the adaptive median filter with maximum size of 5×5 on the image in JPEG format. Some noises still remain in the image. (c) The intensity distribution of the image in TIFF format. (d) The intensity distribution of the image in JPEG format.

Our motivation is to design a filter that efficiently removes impulse noises with minor variations while keeping the non-noisy pixels unaltered. When impulse noises have minor variations, adaptive median filters can not remove the noises cleanly and the standard median filter ruins the fine details. The proposed filter introduced in the next section classifies noises and non-noisy pixels with high accuracy even though the impulse noises have minor variations and therefore details can be well preserved.

The remainder of the paper is organized as follows: Section (2) introduces the new filter and analyze the mathematical mechanism behind it. Section

(3) demonstrates the effectiveness of the new filter by comparing with the standard median filter and the adaptive median filter used in [3]. Section (4) is the summary and conclusions.

2 Sorted difference filter

For a pixel (x, y) in a given image, denote

$I(x, y)$ the intensity at the pixel (x, y), and
S_{xy} a neighborhood centered at (x, y).

Suppose S_{xy} is encompassed by a filter centered at (x, y) with a size $m \times n$. For every pixel $(x', y') \in S_{xy}$ we compute the absolute value of the intensity difference between the pixel (x', y') and the center pixel (x, y),

$$d_{x'y'} = |I(x, y) - I(x', y')|,$$

and call it the *absolute difference* for the pixel (x', y'). Denote D_{xy} the sorted sequence of all the absolute differences found in S_{xy} in ascending order,

$$D_{xy} = \text{ the sorted sequence of } \{d_{x'y'} | (x', y') \in S_{xy}\}.$$

Let l be the length of the sorted sequence D_{xy}, then $l = mn$. We use an index i to access a particular value in D_{xy}, then

$$D_{xy} = \{D_{xy}[i]\}_{i=0}^{l-1}.$$

It is easy to see that $D_{xy}[0] = 0$ and $D_{xy}[i]$ is non-decreasing in i.

We use a threshold T on a specific value $D_{xy}[i^*]$, $0 \leq i^* < l$, to select noise candidates. The threshold T is an experimental value that can be adjusted in applications, while the index i^* is mainly determined by the size of the filter and the density of the impulse noises in the image.

The threshold T is selected in such a way that noises with absolute differences higher than T can be selected out. To determine i^*, suppose the densities of salts and peppers are both p, then empirically there are about pl salts and pl peppers in S_{xy}. Let i^* be the nearest whole number no less than pl, giving by the ceiling function

$$i^* = ceiling(pl). \tag{3}$$

The index i^* is the property of the filter and it applies to all the pixels under processing. Once i^* is determined, it does not change during the processing.

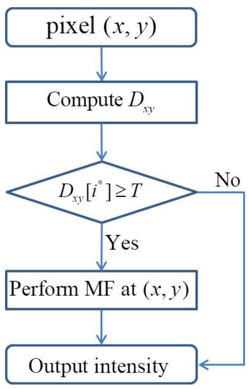
If (x, y) is a noise, say a salt, because we expect there are i^* salts in S_{xy}, then for $0 \leq i < i^*$, $D_{xy}[i] = 0$ if the impulse distribution has no variations, or $D_{xy}[i] < T$ for a proper threshold T if the impulse distribution has minor variations. The value $D_{xy}[i^*]$ is the first absolute difference between the salt noise and the background with an abrupt increment. If $D_{xy}[i^*] \geq T$ then the salt noise can be singled out. The situation is the same when (x, y) is a pepper.

For every pixel (x, y), let n_{xy} be the number of pixels in S_{xy} such that the absolute differences are less than the threshold T. Obviously $D_{xy}[i] < T$ for $0 \leq i < n_{xy}$ and $D_{xy}[n_{xy}] \geq T$. From the above analysis, if (x, y) is a noise, then statistically $n_{xy} \leq i^*$. If (x, y) is a non-noisy pixel and $n_{xy} > i^*$ then the pixel can be easily differentiated from a noise.

The number n_{xy} changes with the pixel location. The filter is based on that the following condition can be reached for a general non-noisy pixel:

$$n_{xy} > i^*. \tag{4}$$

We will see that the condition is generally satisfied if the noise density is not too heavy. For every pixel (x, y) find the sequence D_{xy} in its neighborhood S_{xy} encompassed by the filter. If $D_{xy}[i^*] \geq T$ then (x, y) is regarded as a noise and its value is replaced by the median intensity value in S_{xy}; otherwise, the pixel is treated as a non-noise pixel and its value is unchanged.



The objective of the method is to apply the standard median filter only for the impulse noises and keep the non-noisy pixels unaltered. The effect of the method is determined by the correctness of classifying noises and non-noisy pixels.

For a non-noisy pixel (x, y), there are a few factors affecting n_{xy}. If the absolute differences of all non-noisy pixels in S_{xy} are less than T and the absolute differences of all noises in S_{xy} are not less than T, which usually occurs when (x, y) is in a region with slow intensity changes, then n_{xy} is approximately equal to $l - 2pl$ because there are about $2pl$ noises in S_{xy}. We use the closest whole number that is no more than $l - 2pl$ for n_{xy} in this case,

$$n_{xy} = floor(l - 2pl). \tag{5}$$

For condition (4) to be satisfied, there should be

$$l - 2pl > pl, \tag{6}$$

which yields $p < 1/3$. This means when the impulse noise density is not heavy, a non-noisy pixel can be

correctly identified in general. The filter is equivalent to the standard median filter if the densities of both salts and peppers are not less than 1/3, or the overall density of the impulses is not less than 2/3.

Notice that the non-noisy pixel (x, y) could be on an edge in the image. If (x, y) is on the boundary between two regions with certain sizes and the filter size is relatively small so the boundary is approximately straight, then generally there are at least half of the non-noisy pixels in the same region of (x, y). Suppose the fraction of such pixels in the same region is q and the intensity difference between the two regions is not less than T, then $n_{xy} = (l - 2pl)q$ approximately. We use

$$n_{xy} = \mathit{floor}\left((l - 2pq)q\right). \tag{7}$$

To keep the edge sharp, the pixel (x, y) should not be altered. This requires that the inequality (4) holds, which is

$$(l - 2pl)q > pl, \tag{8}$$

giving

$$p < q/(1 + 2q). \tag{9}$$

Using the empirical value $q \approx 0.5$, we get $p < 0.25$. As mentioned in [3], the standard median filter performs well when $p < 0.2$. Under the same condition, the new filter works better on edge preserving than the standard median filter because edge pixels are not altered.

Theoretically, if the threshold T is properly selected and the densities of the salts and peppers are not heavy, for example if they are less than 0.25, the noises and the non-noisy pixels can be recognized correctly by the new filter. The filter checks $D_{xy}[i^*] \geq T$ to determine whether a standard median filter should be used at (x, y). The threshold T can be adjusted in processing to get an optimal result. When $T = 0$ then $D_{xy}[i^*] \geq T$ is always true so the new filter is exactly the same as the standard median filter. When T is large enough such that $D_{xy}[i^*] \geq T$ can never be satisfied then the new filter does not change anything of the image.

Practically, even though the density p is known beforehand the index i^* may not work well to remove all the noises. This is because digital images are discretely represented and noises are not perfectly evenly distributed, and therefore noises may form clusters with sizes larger than i^*. If a noise (x, y) is in such a cluster and there are more than i^* pixels of the cluster in S_{xy}, then $D_{xy}[i^*]$ is very small and it cannot be detected by the threshold. To remove noise clusters the threshold should be applied to $D_{xy}[j]$ with a bigger index $j > i^*$. Such an index j must be no bigger than n_{xy}, otherwise non-noisy pixels cannot be correctly recognized.

Generally, when the noise density is not heavy there is a big gap between i^* and n_{xy} for every non-noisy pixel (x, y). This means a proper index j can be easily selected such that $i^* < j \leq n_{xy}$. For example, when $p = 0.1$ and the size of the filter is 5×5, which gives $l = 25$, then by (3) $i^* = 3$. To compute n_{xy}, we choose (7) with $q = 0.5$ because the resultant value is smaller than the value given by (5), and get $n_{xy} = (l - 2pl)q = 10$. Then j can be selected in the range $3 < j \leq 10$. The gap between i^* and n_{xy} provides flexibility for the filter to find a proper index j in applications without knowing the actual density of the impulses. Notice that a big j also remove some tiny components of objects in an image.

In summary, for a given image corrupted by impulses with minor variation, select a filter size $l = m \times n$ and set up initial values of the index j and the threshold T. If the impulse density p can be obtained then j can be initially chosen from $lp < j < (l - 2pl)q$ with $q = 0.5$; if not, j can still be easily selected because of the big gap between i^* and n_{xy}. For every pixel (x, y) under processing, sort the absolute differences of pixels in the neighborhood encompassed by the filter in ascending order to get the sequence D_{xy}. If $D_{xy}[j] \geq T$, then the standard median filter with the same filter size is applied; otherwise, the intensity value at (x, y) is unchanged. Finish processing the image to get a result. Adjust the threshold T with each j, and also adjust j and the size of the filter as needed until an optimal result is obtained.

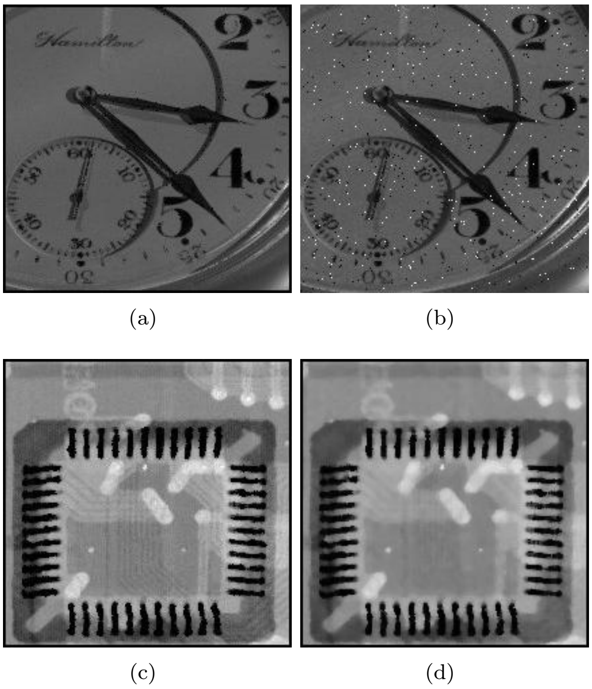
3 Experimental results

In our experiments, the new filter is compared with the standard median filter and the adaptive median filter in ([3]).

The top two images in Figure 3 (a) and (b) display the results of the new filter and the adaptive median filter on the test image in Figure 1 (a), respectively. The image in Figure 3 (a) is obtained by the new filter with a size 5×5, $j = 8$, and $T = 40$. Compared with the image in Figure 1 (b) obtained by the standard median filter, the new result is much perceptually better because it keeps more details of the watch. The Peak Signal to Noise Rate (PSNR) of the image in Figure 3 (a) is 30.31, which is apparently higher than the PSNR of the image in Figure 1 (b), 27.76. Notice that RSNR is only a coarse estimate of the effectiveness of a denoising method, and the perceptual effect of the method is also an important consideration. The image in Figure 3 (b) is the result obtained by the adaptive median filter. Because the impulse noises have minor variations some noises are not cleanly removed. The PSNR of this resultant image is 23.86 owing to the remaining peppers and salts.

At the bottom of Figure 3, results with two different methods on the test image in Figure (2) (a) are displayed. With the new filter, by setting the filter size 5×5, $j = 10$ and threshold $T = 22$, we get the result shown in Figure 3 (c). For comparison, the result

obtained by the standard median filter with the same size is also displayed in Figure 3 (d). Perceptually, the fine details are kept very well in the image obtained by the new filter but they are severely blurred by the standard median filter. The PSNRs are 25.31 for image in Figure 3 (c) and 24.74 for the image in Figure 3 (d), with the new filter giving the higher one. Because the impulse noises have variations on their intensities, some noises cannot be removed by the adaptive median filter, shown in Figure 2 (b), which is not acceptable because of the remaining noises.



(a) (b) (c) (d)

Figure 3: (a) The result obtained by the new filter on the test image in Figure 1. (b) The result obtained by the adaptive median filter. (c) The result of the new filter on the test image in Figure 2 (a). (d) The result by the standard median filter with size 5×5.

Next, we display the comparisons on the commonly used test images Barbara, Boat, Cameraman and Lena. For each test image, we added impulsive salt and pepper noises by 10%, 20%, 30% and 40%, respectively, and then converted the noisy images into JPEG format. Then the adaptive median filter (AMF), the standard median filter (MF) with size 5×5, and the new filter (NF) with size 5×5, a proper index j and a threshold T were applied on these images. Images with 20% salt and pepper noises and the corresponding filtered images by the mentioned filters are shown in Figures 5 through 8. All PSNR comparisons are summarized in a table at the end. Because the test images with noises are saved in JPEG format, the noise distributions all have minor variations, shown at the two ends of the intensity range of each distribution chart in Figure 4.

Figure 5 displays the comparisons on the noise corrupted image Barbara in JPEG format. The top left image is the test image corrupted with 20% salt and pepper noise and the top right is the result by NF. The bottom left is the result by AF and the bottom right is the result by MF. Among the three filtered images, the image by NF shows the best perceptual effect.

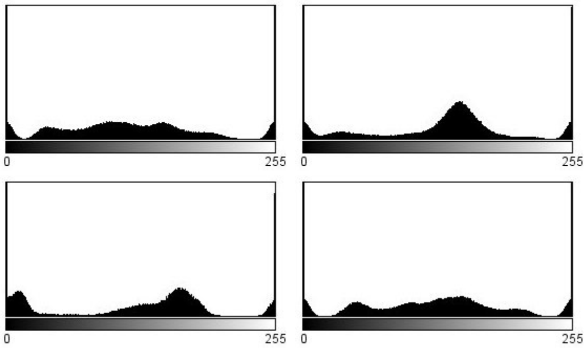


Figure 4: Intensity distributions of the test images corrupted with 20% salt and pepper noise, saved in JPEG format. Top left for Barbara; Top right for Boat; Bottom left for Cameraman; Bottom right for Lena.

Figure 5: Top left, the noise corrupted image. Top right, the result by the new filter. Bottom left, the result by the adaptive median filter. Bottom right, the result by the standard median filter.

Figures 6, 7 and 8 display similar comparisons for the noise corrupted test images Boat, Cameraman and Lena.

Among the above images, the results obtained by the new filter preserve more fine details than the results obtained by the standard median filter. The results obtained by the adaptive median filter still contain noticeable noises so they are not of satisfactory.

Figure 6: Top left, the noise corrupted image. Top right, the result by the new filter. Bottom left, the result by the adaptive median filter. Bottom right, the result by the standard median filter.

Figure 7: Top left, the noise corrupted image. Top right, the result by the new filter. Bottom left, the result by the adaptive median filter. Bottom right, the result by the standard median filter.

The PSNRs of the filtered images for the test images corrupted with different percentages of salt and pepper noises are listed in the following table. For a test image corrupted with noise less than 40%, the PSNR of the result obtained by the new filter is the highest. When the noise percentage is higher than 40%, the effect of the new filter is essentially the same as that of the standard median filter.

Figure 8: Top left, the noise corrupted image. Top right, the result by the new filter. Bottom left, the result by the adaptive median filter. Bottom right, the result by the standard median filter.

Image	Noise percentage	PSNR for different filters		
		AMF	MF	NF
Barbara	10%	22.75	23.04	24.02
	20%	18.93	22.88	23.02
	30%	15.75	22.65	22.68
	40%	13.72	22.15	22.15
Boat	10%	23.78	26.87	27.86
	20%	19.35	26.58	27.24
	30%	15.95	25.90	26.19
	40%	13.93	25.04	25.10
Cameraman	10%	23.12	23.50	24.65
	20%	18.80	23.29	23.71
	30%	15.38	22.77	22.83
	40%	13.31	22.11	22.05
Lena	10%	24.06	30.33	32.00
	20%	19.43	29.90	30.57
	30%	16.02	29.06	29.22
	40%	13.95	27.75	27.76

4 Conclusion

A novel method is proposed to eliminate impulse noises with minor variations. The method uses a filter to find out impulse noises and replace them with the median intensity values in their neighborhoods, while the non-noisy pixels are not altered. The proposed filter outperforms the standard median filter in fine details preserving because only the noises are processed. The new filter is immune to minor variations of the impulses, so it is more applicable than the adaptive median filter that works well on details preserving and noises elimination only when the impulses have no variations. Theoretically, when the density of the impulses is not heavy the noises can be correctly

identified. The parameters used in the method can also be easily adjusted in applications to obtain optimal results.

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